A Mathematical Programming Approach for Pilot Power Optimization in WCDMA Networks

Peter Värbrand and Di Yuan

Department of Science and Technology, Linköping University
SE-601 74 Norrköping, Sweden
petva, diyua@itn.liu.se

Abstract – Pilot power optimization in WCDMA networks is receiving increasing research attention. In this paper, we propose a mathematical programming approach for this problem. In particular, we present a novel mathematical formulation for minimizing pilot power subject to full coverage. In the formulation, the optimal power levels are determined simultaneously for all the cells. We further enhance the formulation by utilizing the discrete nature of the power levels. Our case study shows that optimized power levels yield substantial savings in total power consumption when compared to using a uniform pilot power.

I. Introduction

Pilot power control and optimization is a crucial engineering issue in Wideband Code Division Multiple Access (WCDMA) networks. The problem of choosing the optimal levels of the pilot power is a challenging task, which involves the trade-off between full coverage on one hand, and the power consumption on the other hand. In this paper, we address the problem of guaranteeing full coverage with a minimum amount of power for the pilot signals.

In a WCDMA network, a pilot signal is associated with each cell, and is used to provide channel estimation to the mobile terminals for cell selection and handover. A mobile terminal measures and compares the pilot signals that it can detect, and is typically attached to the cell with the best quality of the pilot signal. Factors that determine the level of the received pilot signal include the transmitting power used for the signal, the signal attenuation between the base station and the mobile terminal, and the effect of the thermal noise. The power of the pilot signal influences the coverage area of the cell, and can therefore be used to balance the traffic load between cells.

In a simple propagation scenario, where the signal attenuation is essentially determined by distance, the mobile terminal will be attached to a cell that belongs to the closest base station, íf all the cells use the same level of pilot power. With fairly uniformly distributed traffic and equally-spread base stations, the sizes of the cells will be roughly the same.

However, in an in-homogenous planning situation (e.g., a mix of rural and downtown areas), a uniform level of pilot power is not an efficient solution from the power consumption point of view. There is therefore a need of mechanisms that can determine the optimal levels of the pilot power.

There is an increasing interest in the literature for control and optimization of pilot signal. In [6], for example, a cost-minimization method is used in network simulations. Based on target values for coverage and traffic load, the method attempts to minimize the deviation from the target values by adjusting the levels of the pilot power using a gradient decent procedure. A similar approach is used in [7]. The authors of [4] consider the problem of pilot power minimization subject to coverage constraints (similar to the problem studied in this paper), and present a (heuristic) iterative method that adjusts the pilot power for one cell in each iteration. A rule-based optimization of the pilot powers is embedded into a simulation tool in [5], where the authors obtain results that outperform manually-designed solutions both in terms of deployment time and cost. The authors of [8] and [9] consider power management for load balancing, and show that network performance can be enhanced by proper adjustments of the pilot powers.

In this paper, we propose a mathematical programming approach for pilot power optimization. We consider the problem of minimizing the total amount of pilot power subject to full coverage, and present a linear-integer formulation, where the levels of the pilot power are simultaneously optimized for all the cells, thus avoiding sub-optimal solutions that may occur when the pilot power is optimized for one cell at a time. Our case study shows that, compared to using a uniform level of pilot power, optimized pilot powers yields substantial savings in the power consumption.

The remainder of this paper is organized as follows. In Section II we describe our system model. The optimization problem is formalized in Section III. We develop the mathematical models in Section IV, and present a case study in Section V. Finally, in Section VI we draw some conclusions and outline some directions for future research.

II. System Model

Consider a WCDMA network with m cells, and let P_i^{Tot} be the total transmission power available in cell i. This power can be allocated to the pilot channel and other signaling channels, as well as user traffic. For cell i, we use x_i to denote the amount of power allocated to the pilot signal.

Clearly, a higher value of x_i means less power available to provide services in cell i.

In planning pilot powers, predictions of signal propagation are carried out. Typically, this prediction is done over a grid of bins (with a certain resolution) covering the entire service area. Let n be the total number of bins, and g_{ij} be the power gain between the base station of cell i and bin j. Thus, in bin j the power of the received pilot signal of cell i is $g_{ij}x_i$. Here, we assume that the conditions of signal propagation are the same across the entire bin (otherwise the resolution of the grid can be increased to produce a more accurate prediction).

In addition to the pilot signal from cell i, bin j also receives interfering signals, including the signals for user traffic from cell i, and signals from some other base stations. The total interference for bin j can be written as

$$I_{ij} = \alpha(P_i - x_i)g_{ij} + \sum_{k \neq i} P_k g_{kj} + \nu_j, \qquad (1)$$

where $\alpha \in (0,1)$ is the orthogonality to signals from the base station of i, P_k is the power level used in cell k (for the pilot signal and user traffic in k) , and ν_j is the effect of the thermal noise in bin j,

We will consider network scenarios with high traffic loads. In particular, we assume that all the base stations operate at full power, i.e., the amount of power consumed for supporting user traffic is $P_i^{Tot} - x_i$ for cell i. Under this assumption, $P_i = P_i^{Tot}, \forall i = 1, \ldots, m$, and (1) thus becomes

$$I_{ij} = \alpha(P_i^{Tot} - x_i)g_{ij} + \sum_{k \neq i} P_k^{Tot} g_{kj} + \nu_j. \qquad (2)$$

For bin j, the quality of the pilot signal of cell i is measured using the carrier-to-interference ratio (CIR), which is defined as

$$\gamma_{ij} = \frac{g_{ij}x_i}{I_{ij}} = \frac{g_{ij}x_i}{\alpha(P_i^{Tot} - x_i)g_{ij} + \sum_{k \neq i} P_k^{Tot} g_{kj} + \nu_j}. \qquad (3)$$

We assume that the pilot signal of cell i can be detected in bin j if and only if the CIR is above a threshold γ_0, that is, if

$$\gamma_{ij} \geq \gamma_0. \qquad (4)$$

Increasing the pilot power yields better coverage. However, at high traffic loads this will reduce the power that can be used for traffic channels. The optimization problem presented in the next section is motivated by this interesting trade-off – to ensure full coverage in the network with the smallest amount of pilot power (i.e., most power will be available for traffic channels).

III. The Optimization Problem

A. Problem definition

We consider the problem of minimizing the total power of pilot signals, such that each bin is covered by a least one cell. The definition of the optimization problem is as follows.

Input

- A number of cells $1, \ldots, m$.
- A number of bins $1, \ldots, n$.
- The total power available in each cell, P_i^{Tot}, $i = 1, \ldots, m$.
- The interference level for each pair of cell and bin, i.e., $I_{ij}, i = 1, \ldots, m, j = 1, \ldots, n$.
- An upper bound of the pilot power in cell i, $P_i^0, i = 1, \ldots, m$. (Parameter P_i^0 can be used to limit the pilot power from above, if this is desired. Otherwise we can set $P_i^0 = P_i^{Tot}$).

Objective

- Minimize the total amount of power used for the pilot signals, i.e., $\min \sum_{i=1}^m x_i$.

Constraints

- For every bin j, there exists at least one cell i for which $\gamma_{ij} \geq \gamma_0$.
- The pilot power of cell i is limited by P_i^0, i.e., $x_i \leq P_i^0, i = 1, \ldots, m$.

We use MPP to denote this optimization problem.

The theoretical computational complexity of MPP is formalized in the following proposition.

Proposition MPP is $\mathcal{NP}$-hard.

Proof See Appendix I.

B. Uniform pilot power

Instead of minimizing the total amount of pilot power, one alternative objective function is to minimize the highest level of the pilot signals, that is,

$$P^U = \min \max_{i=1,\ldots,m} x_i. \qquad (5)$$

However, we observe that (5) can be calculated analytically. Specifically, using (3) and (4), we compute

$$P_{ij} = \frac{\gamma_0(\sum_{k \neq i} P_k^{Tot} g_{kj} + \nu_j) + \gamma_0 \alpha P_i^{Tot} g_{ij}}{(1 + \gamma_0\alpha)g_{ij}}. \qquad (6)$$

Clearly, P_{ij} is the minimum level of pilot power for cell i to cover bin j. Now let

$$P_j^U = \min_{i=1,\ldots,m} P_{ij}. \qquad (7)$$

To cover bin j, the power of at least one pilot signal in the network must be greater than or equal to P_j^U. Consequently, if we take the maximum of P_j^U over all the bins, we obtain the solution to (5):

$$P^U = \max_{j=1,\ldots,n} P_j^U. \qquad (8)$$

In fact, it can be easily realized that if all the cells use a uniform pilot power, this uniform pilot power is exactly P^U for providing full coverage.

IV. Mathematical Formulations

A. A cell-bin formulation

In addition to the power variables $x_i, i = 1, \ldots, m$, we use the following binary variable for every pair of cell and bin.

$$y_{ij} = \begin{cases} 1 & \text{if cell } i \text{ covers bin } j, \text{(i.e., } \gamma_{ij} \geq \gamma_0\text{)}, \\ 0 & \text{otherwise.} \end{cases}$$

MPP can be formulated using the following cell-bin formulation.

$$\text{[MPP-CB]} \;\; P^* = \min \sum_{i=1}^{m} x_i \tag{9}$$

$$\sum_{i=1}^{m} y_{ij} \geq 1, \;\; j = 1, \ldots, n, \tag{10}$$

$$P_{ij} y_{ij} \leq x_i, \;\; i = 1, \ldots, m, j = 1, \ldots, n, \tag{11}$$

$$y_{ij} = 0, \quad i = 1, \ldots, m, \; j = 1, \ldots, n : P_{ij} > P_i^0, \tag{12}$$

$$y_{ij} \in \{0, 1\}, \; i = 1, \ldots, m, j = 1, \ldots, n. \tag{13}$$

In MPP-CB, constraints (10) ensure that every bin is covered by at least one cell. By constraints (11), pilot power x_i must be at least P_{ij}, if cell i covers bin j. If the power needed to cover bin j exceeds the power limit at cell i, then obviously cell i cannot cover bin j (i.e., $y_{ij} = 0$). This is stated in constraints (12). (In fact, such y-variables can be eliminated from the formulation in a preprocessing phase.) Note also that the non-negativity restrictions on the x-variables are implicitly handled by (11).

B. An Enhanced Formulation

MPP-CB is a quite straightforward linear integer program. However, this formulation is not efficient from a computational point of view. In particular, the linear programming (LP) relaxation of MPP-CB is very weak (i.e., the LP-optimum is far away from the integer optimum), making the problem intractable for a standard branch-and-bound enumeration solution technique. In our case study, a state-of-the-art integer programming solver [3] did not manage to find optimal or near-optimal solutions within any reasonable amount of time for a network with 60 cells.

We present an enhancement to MPP-CB that substantially improves the LP-relaxation. We utilize the fact that in an optimal solution to MPP, the pilot power of any cell will attain a value that belongs to a discrete set. That is, in an optimal solution $x_i = P_{ij}$ for some bin j (because there is no need to increase the level of the pilot power unless additional bins can be covered).

We introduce the following notation. For cell i, we sort P_{ij} in ascending order, and use $b_1, b_2, \ldots, b_{l-1}, b_l, \ldots$, to denote the sorted indices of bins. That is, $P_{ib_1} \leq P_{ib_2} \leq \ldots \leq P_{ib_{l-1}} \leq P_{ib_l} \leq \ldots$, where b_l is the bin that appears at position l in the sorted sequence. We use B_i to denote the total number of bins in the sorted sequence. (Usually $B_i < n$, because bins for which the pilot power exceeds P_i^0 can be ignored.) Moreover, we consider the sorted sequence of power levels in an incremental fashion:

$P_{ib_1}^I = P_{ib_1}$,
$P_{ib_2}^I = P_{ib_2} - P_{ib_1}$,
$\ldots$,
$P_{ib_l}^I = P_{ib_l} - P_{ib_{l-1}}$,
$\ldots$,
$P_{ib_{B_i}}^I = P_{ib_{B_i}} - P_{ib_{B_i-1}}$.

For any index $l \in [1, B_i]$, the value of $P_{ib_l}^I$ is the additional power needed for cell i to cover bin b_l. The pilot power of cell i is thus $\sum_{l=1}^{B_i} P_{ib_l}^I y_{ib_l}$. The total amount of pilot power can therefore be expressed as

$$P^* = \sum_{i=1}^{m} \sum_{l=1}^{B_i} P_{ib_l}^I y_{ib_l}. \tag{14}$$

Using (14) to formulate the objective function, the constraints (11) (which define the pilot powers) are no longer needed. In addition, constraints (12) can be handled when defining the sorted power sequences for the cells.

Note that if cell i covers bin b_l, then obviously the cell also covers bins $b_1, b_2, \ldots, b_{l-1}$. This fact is formulated using the constraints below.

$$y_{ib_l} \leq y_{ib_{l-1}}, \;\; l = 2, \ldots, B_i, i = 1, \ldots, m. \tag{15}$$

Below we summarize the enhanced version of MPP-CB.

$$\text{[MPP-CBE]} \;\; P^* = \min \sum_{i=1}^{m} \sum_{l=1}^{B_i} P_{ib_l}^I y_{ib_l} \tag{16}$$

$$(10), (15), \text{ and } (13).$$

The enhanced formulation, MPP-CBE, is much more efficient than MPP-CB, in terms of the solution quality of the LP-relaxation. This is formalized in the following proposition.

Proposition The LP-relaxation of MPP-CBE is at least as strong as the LP-relaxation of MPP-CB. In addition, there exist instances for which the former is strictly better than the latter.

Proof See Appendix II.

For the network used in our case study, the relative gap between the LP-optimum and the integer optimum is more than 50% for MPP-CB. Using MPP-CBE, the gap is reduced to less than 1%, and the integer optimum could be found and verified in less than one second (on a computer with a 400 MHz UltraSparc CPU).

V. A Case Study

The scenario in our case study has 22 base stations and 60 cells. The scenario represents a real network being planned. The locations of the base stations are restricted to positions where building permits are granted, and are consequently not optimized from the coverage point of view. It is therefore important to optimize other system parameters, such as the pilot power.

The service area consists of 1440 bins, and among them 1375 bins are to be covered. Each bin has a size of 40×40 meters. Parameter P_i^{Tot} is the same for all the cells. In addition, all the bins have the same level of thermal noise. We use (8) (see Section III-B) to compute the minimum uniform pilot power P^U, and found that $P^U = 1.087$. A summary of the problem parameters is shown in Table I.

TABLE I

PARAMETER SETTING.

Parameter	m	n	P_i^{Tot}	γ_0	ν_i	α	P^U
Value	60	1375	15 W	0.015	10^{-13}	0.6	1.0873

The formulation MPP-CBE was solved using AMPL [1] and CPLEX 7.0 [3]. We consider two different settings for P_i^0 (the upper limit of the pilot power). In the first setting, $P_i^0 = 2$ W, $i = 1, \ldots, m$. In the second setting, we set $P_i^0 = P^U, i = 1, \ldots, m$. Using the former setting yields the minimum total pilot power, but there are some cells whose pilot powers exceed P^U. Using the latter setting, no cell will use more than P^U in the pilot power. However, the total pilot power becomes slightly higher. The results are displayed in Table II.

TABLE II

RESULTS OF PILOT POWER OPTIMIZATION.

	Total	Average
Uniform pilot power P^U	65.2384	1.0873
Optimized pilot power, $P_i^0 = 2$	27.8689	0.4645
Optimized pilot power, $P_i^0 = P^U$	28.0780	0.4680

We observe that solving MPP yields substantial savings in the pilot power. In particular, the average pilot power of a cell is less than 0.5 W, which corresponds to a saving of more than 50% when compared to P^U. In addition, the total pilot power differs by less than 1% for the two settings of P_i^0.

The optimized pilot powers are further studied in Figures 1 and 2. The two figures show a histogram of the pilot powers, and the power distribution, respectively. It can be observed that very few cells use a pilot power higher than 1.0 W in the optimized solutions. In fact, for a majority (over 90%) of the cells the pilot power is less than 0.9 W. The pilot power has a similar distribution for the two different settings of P_i^0. However, when it is more desired to limit the power of any individual cell, it is more preferable to set $P_i^0 = P^U, \forall i$.

Figure 3 shows a histogram of the CIR of the bins. If the uniform pilot power is used, the CIR values of most bins are greater than $5\gamma_0$, i.e., there are many bins for which the margin between the CIR and γ_0 is unnecessarily large. When the pilot powers are optimized, unnecessarily high CIR is avoided, and most of the bins are covered with CIR between γ_0 and $4\gamma_0$.

VI. CONCLUSIONS AND FUTURE RESEARCH

We proposed a mathematical programming approach for pilot power optimization in WCDMA networks. The

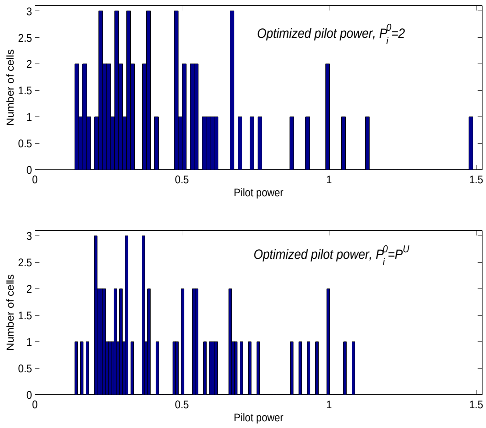

Fig. 1. A histogram of the pilot power.

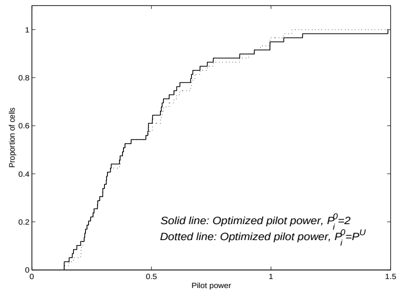

Fig. 2. Pilot power distribution.

optimization problem concerns minimizing the total pilot power subject to full coverage. The formulation was then enhanced by a discretization of the power levels. In our case study, the optimized pilot powers substantially outperform the solution of using the uniform pilot power, indicating that the proposed framework of methodology can be very useful for enhancing power efficiency for WCDMA networks.

There are several interesting topics for future research. In particular, we note that the size of MPP-CBE increases rapidly with respect to the number of bins. For large-scale scenarios, therefore, it may become unfeasible to solve MPP-CBE using a standard branch-and-bound enumeration, and other solution techniques, such as decomposition schemes, need to be developed. We are currently investigating a solution technique based on a Lagrangian relaxation for large-scale instances of MPP. Another important research topic is to consider traffic scenarios where the total power consumed by some cells (for the pilot signal and user traffic) is less than the upper limit, i.e., $P_k < P_k^{Tot}$ for some k. For such scenarios, pilot power optimization for the purpose of load balancing leads to more complex optimization problems, for which models and solution methods need to be developed.

VII. ACKNOWLEDGMENT

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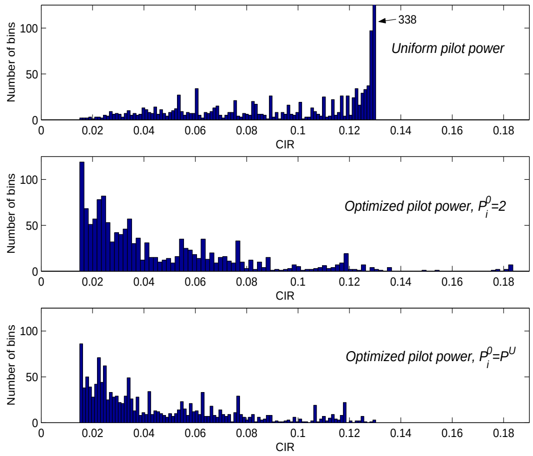

Fig. 3. A histogram of carrier-to-interference ratio.

sions and the test data. This work is partially financed by CENIIT (Center for Industrial Information Technology), Linköping Institute of Technology, Sweden.

References

[1] R. Fourer, D. M. Gay and B. W. Kernighan, *AMPL - A Modeling Language for Mathematical Programming.* Boyd & Fraser, Danvers, MA, 1993.

[2] F. *Gunnarsson, Pilot Power Setting in an in-homogenous wireless network*, Research PM, Ericsson Radio Systems, 2002.

[3] ILOG, "Ilog cplex, user's manual," August 2000.

[4] D. Kim, Y. Chang, and J. W. Lee, "Pilot Power Control and Service Coverage Support in CDMA Mobile Systems," *Proceedings of IEEE Vehicular Technology Conference (VTC '99)*, pp. 1464-1468, 1999.

[5] R. T. Love, K. A. Beshir, D. Schaeffer, and R. S. Nikides, "A pilot optimization technique for CDMA cellular systems," *Proceedings of IEEE Vehicular Technology Conference (VTC '99)*, pp. 2238-2242, 1999.

[6] K. Valkealahti, A. Höglund, J. Pakkinen, and A. Flanagan, "WCDMA Common Pilot Power Control with Cost Function Minimization," *Proceedings of IEEE Vehicular Technology Conference (VTC Fall '02)*, pp. 2244-2247, 2002.

[7] K. Valkealahti, A. Höglund, J. Pakkinen, and A. Flanagan, "WCDMA Common Pilot Power Control for Load and Coverage Balancing," *Proceedings of Fifth IEEE International Symposium on Wireless Personal Multimedia Communication (PIMRC '02)*, pp. 1412-1416, 2002.

[8] J. Yang and J. Lin, "Optimization of pilot power management in a CDMA radio network," *Proceedings of IEEE Vehicular Technology Conference, VTC Fall '00* pp. 2642-2647, 2000.

[9] H. Zhu, T. Buot, R. Nagaike, and S. Harman, "Load Balancing in WCDMA Systems by Adjusting Pilot Power," *Proceedings of IEEE Vehicular Technology Conference, VTC Fall '02*, pp. 936-940, 2000.

Appendix I
Proof of Proposition 1

Proof: We show that any instance of the minimum-cost set covering problem (which is $\mathcal{NP}$-hard) can be transformed to an instance of MPP. Consider an instance of the set covering problem, where $\{S_1, S_2, \dots, S_m\}$ is collection of sets, and B is a set of items. Each set S_i is associated with a cost c_i. In addition, let an indication parameter a_{ij} be 1 if set S_i contains item j, and zero otherwise. The objective of the set covering problem is to select a subset of $\{S_1, S_2, \dots, S_m\}$ at minimum cost, such that all the items in B are covered by at leat one set. The corresponding instance of MPP has m cells and $m + |B|$ bins. In addition, we choose the parameters P_i^{Tot}, α, g_{ij}, ν_j, P_i^0, and γ_0 such that the following hold.

- For bin $j = 1, \dots, m$, $P_{jj} = \epsilon$, where ϵ is a positive number that is less than $\min_{i=1,\dots,m} c_i$, and $P_{ij} > P_i^0, \forall i \neq j$. (That is, cell j is the only cell that can cover bin j.)
- For cell $i = 1, \dots, m$ and $j = m + 1, \dots, m + |B|$, $P_{ij} = c_i$ if $a_{ij} = 1$, otherwise $P_{ij} > P_i^0$.

It can be easily realized that choosing a parameter setting with the above properties can be down in polynomial time. Moreover, a feasible solution in the MPP instance is also feasible in the set covering instance, and vice versa. Moreover, for any such pair of solutions, the two objective functions have the same value. Hence the conclusion. ■

Appendix II
Proof of Proposition 2

Proof: The first part of the proposition can be proved as follows. Given any feasible solution in the LP-relaxation of MPP-CBE, we observe that

1) this solution is also feasible in the LP-relaxation of MPP-CB, and
2) the objective function value of MPP-CBE for this solution is greater than or equal to that of MPP-CB.

Given a feasible solution, denoted by $\bar{y}_{ij}, \forall i, j$, to the LP-relaxation of MPP-CBE, it can be easily realized that, because $\bar{y}$ satisfies (10), it is also feasible in the LP-relaxation of MPP-CB. Hence the conclusion in 1). For $\bar{y}$, the optimal solution of x_i in MPP-CB is obviously $\bar{x}_i = \max_{j=1,\dots,n} P_{ij}\bar{y}_{ij}$. Assume that the maximum occurs for bin j^*, i.e., $\bar{x}_i = P_{ij^*}\bar{y}_{ij^*}$. Consider MPP-CBE, and the sorted sequence $b_1, \dots, b_l, \dots, b_{B_i}$. Without any loss of generality, assume that $b_{l^*} = j^*$. The pilot power of bin i in MPP-CBE equals

$$\sum_{l=1}^{B_i} P^I_{ib_l}\bar{y}_{ib_l} \geq \sum_{l=1}^{l^*} P^I_{ib_l}\bar{y}_{ib_l} \geq \bar{y}_{ib_{l^*}} \sum_{l=1}^{l^*} P^I_{ib_l} = \bar{y}_{ib_{l^*}} P_{ib_{l^*}}$$

$= P_{ij^*}\bar{y}_{ij^*}$. (The second inequality is due to (15).) Because above inequalities hold for any i, we have shown 2) and thus the first part of the proposition.

To show the second part of the proposition, it is sufficient to give an example. Consider an instance of MPP with two cells and four bins, where $P_{11} = 1.2, P_{12} = 0.8, P_{13} = 0.6, P_{21} = 0.6, P_{22} = 0.8$, and $P_{24} = 0.3$. Assume also that P_{14} and P_{23} exceed the power limit (and are thus irrelevant to the discussion). In the integer optimum, $y_{11} = y_{12} = y_{13} = y_{24} = 1$, and the total pilot power equals 1.5. The LP-relaxation of MPP-CB yields $y_{11} = 0.5, y_{12} = 0.75, y_{13} = 1, y_{21} = 0.6, y_{22} = 0.25, y_{23} = 1$, and the objective function value equals 0.9. (The relative gap is therefore 40%.) However, the LP-relaxation of MPP-CBE yields the integer optimum. ■