Queueing performance improvement with random order dispersion

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***Abstract*— This paper analytically studies queueing performance improvement with traffic dispersion. We model sources and paths between sources and destinations as generalized binary Markov sources and discrete-time queues, respectively. We suppose that random order dispersion is employed in packet network, and we examine the effect of random order dispersion on the asymptotic tail distribution of the queue length of packets. Numerical results exhibit that traffic dispersion can improve the queueing performance of packets.**

I. Introduction

High-speed networks are expected to integrate a wide variety of applications with different characteristics and quality-of-service (QoS) requirements. In high-speed networks, many applications (e.g., high quality video communications) which recently emerged or will emerge in near future are so called “bandwidth-hungry” and require QoS guarantees. For supporting such applications, which have caused traffic explosion, a kind of load balancing mechanisms is introduced to achieve the effective use of bandwidth while providing QoS guarantees.

One approach to achieve load balancing in packet networks is traffic dispersion (see, e.g., [3], [7] and references therein). With traffic dispersion, a burst of arrivals from a source is divided into many sub-bursts, these sub-bursts are spread over multiple paths (which do not share any physical links) and transmitted in parallel through the network. In other words, traffic dispersion distributes load over multiple paths to avoid situations where a path is heavily loaded in a short time scale. As a result, it is expected that the utilization of traffic dispersion will improve link utilization and/or network performance.

Most studies on the performance evaluation of packet networks have assumed that all packets from a source are transmitted over a single path. This fundamental assumption is violated, when load balancing mechanisms such as traffic dispersion are employed in the network. Thus, for networks where load balancing mechanisms such as traffic dispersion is employed, performance aspects have to be reevaluated.

Several researchers have studied traffic dispersion in packet/cell networks. This generic techniques often appear under many different labels such as multipath routing [10], dispersity routing [11], [2], string mode [5], channel striping or inverse multiplexing [1], [13], parallel communications scheme [4] and connection splitting [3]. Krishnan and Silvester [10] examine the effect of traffic dispersion on the loss performance of cells in ATM networks. They employ MMPP (Markov Modulated Poisson Process) to approximate the superposition of on-off sources and approximately estimate the loss performance of cells by using an analytical result for $\mathrm{MMPP}/\mathrm{D}/1/\mathrm{K}$ queues. They conclude that traffic dispersion offers an order of magnitude performance enhancement with respect to cell loss. Déjean el al. [5] examine the effect of traffic dispersion on the loss performance of cells in ATM networks by simulation. They employ an on-off source model and investigate two dispersion algorithms. The first algorithm is distributing the string, which is a unit of dispersion in their framework, onto a number of links in a random order. The other algorithm is distributing the string in a round-robin manner.

This paper analytically studies the queueing performance improvement with traffic dispersion. We model sources and paths between sources and destinations as generalized binary Markov sources (GBMSs) [12], [8] and discrete-time queues, respectively. We suppose that random order dispersion is employed in the queueing system, and we examine the effect of random order dispersion on the asymptotic tail distribution of the queue length.

The rest of this paper is organized as follows. Section II describes a source model and a queueing model studied in this paper. In Section III, we present the asymptotic analysis of the queueing model. Section IV provides numerical results to examine the effect of traffic dispersion on the tail distribution of the queue length. Conclusion is drawn in Section V.

II. Model

In this section, we first explain a basic source model considered in this paper. We then describe a dispersed source model which is generated from the basic source model by random order dispersion. The dispersed source model falls into the same category of the basic original source model, and the relation between the dispersed source model and the basic original source model is given in term of simple equations.

Before explaining the source models, we state a scenario supposed in this paper.

- When traffic dispersion is employed for a source, traffic generated by the source is equally divided and transmitted over multiple paths which do not share any physical links. More specifically, a path to transmit a packet generated by the source is randomly selected among the multiple paths.

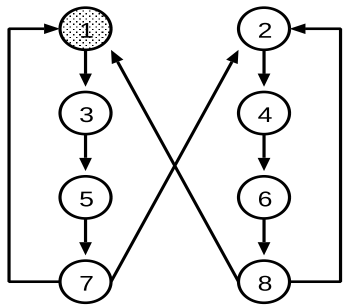

Fig. 1. State transition diagram of GBMS with $R = 4$

In other words, random order dispersion is employed. The number of the multiple paths used for dispersion is called *dispersion factor*.

- In the models considered in this paper, time is divided into equal intervals referred to as slots, and the slot length is equal to a unit of time.

A. Basic source model

In this paper, we consider generalized binary Markov source (GBMS), which is a slight extension of generalized binary Markov source (GBMS) considered in [8], [9], [12], as a basic source model. In fact, when the parameter θ (defined below) of GBMS considered in this paper is equal to 1, the GBMS reduces to GBMS considered in [8], [9], [12].

Before explaining GBMS, we describe binary Markov source (BMS), which is a special case of GBMS. In any slot, BMS is in one of the two different states: on-state and off-state. In off-state, it does not generate a packet, and in on-state, it generates one packet with probability θ in every slot. The transition probability from on-state (resp. off-state) to off-state (resp. on-state) is denoted by $1-\alpha$ (resp. $1-\beta$), where $0 < \alpha, \beta < 1$. The matrix generating function $\hat{\boldsymbol{A}}^*_{\theta,\alpha,\beta}(z)$ for BMS is then given by

$$\hat{\boldsymbol{A}}^*_{\theta,\alpha,\beta}(z) = \begin{pmatrix} \alpha(1-\theta+\theta z) & 1-\alpha \\ (1-\beta)(1-\theta+\theta z) & \beta \end{pmatrix}. \quad (1)$$

Note that BMS is characterized by three parameters G, ρ and B (or θ, α and β), which represents the mean burst size, the average rate and the mean burst length of the BMS, respectively. The following relation holds among those parameters.

$$\alpha = 1 - \frac{1}{B}, \quad \beta = 1 - \frac{\rho}{(\theta-\rho)B},$$
$$G = \theta B, \qquad \rho = \frac{\theta(1-\beta)}{2-\alpha-\beta}.$$

For the BMS with parameters (θ, α, β), the Perron-Frobenius (PF) eigenvalue $\hat{\delta}^*_{\theta,\alpha,\beta}(z)$ of $\hat{\boldsymbol{A}}^*_{\theta,\alpha,\beta}(z)$ is found to be

$$\hat{\delta}^*_{\theta,\alpha,\beta}(z) \;=\; \zeta(z) + \sqrt{\zeta(z)^2 - \kappa(z)}, \quad (2)$$

where

$$\zeta(z) = \frac{\alpha\phi(z)+\beta}{2},$$
$$\kappa(z) = (\alpha+\beta-1)\phi(z),$$
$$\phi(z) = 1-\theta+\theta z.$$

Note that the peak rate of BMS is restricted to one. To overcome this difficulty in modeling, we consider a source model which is called generalized binary Markov source (GBMS). The source is constrained to generate a maximum of one packet every R slots. And in every R slots, it behaves exactly the same as the corresponding BMS. We call this source model a GBMS (generalized BMS). Thus, BMS is a special case of GBMS with $R = 1$. The matrix generating function $\hat{\boldsymbol{A}}_{R,\theta,\alpha,\beta}(z)$ for a GBMS with parameters $(R, \theta, \alpha, \beta)$ is given by

$$\hat{\boldsymbol{A}}_{R,\theta,\alpha,\beta}(z) = \begin{pmatrix} \boldsymbol{O}_{2(R-1),2} & \boldsymbol{E}_{2(R-1),2(R-1)} \\ \hat{\boldsymbol{A}}^*_{\theta,\alpha,\beta}(z) & \boldsymbol{O}_{2,2(R-1)} \end{pmatrix}, \quad (3)$$

where $\boldsymbol{O}_{i,j}$ and $\boldsymbol{E}_{i,i}$ denote an $i \times j$ zero matrix and $i \times i$ identity matrix, respectively, and $\hat{\boldsymbol{A}}^*_{\theta,\alpha,\beta}(z)$ is given by (1). The state transition diagram of GBMS with $R = 4$ is displayed in Fig. 1. In the figure, the shaded state (i.e., state 1) denotes a state where the source generates a packet with probability θ. GBMS is characterized by four parameters G, ρ, B and R (or R, θ, α and β), which represents the mean burst size, the average rate, the mean burst length and the inverse of the peak rate of the GBMS, respectively. The following relation holds among those parameters.

$$\alpha = 1 - \frac{R}{B}, \quad \beta = 1 - \frac{R^2\rho}{(\theta-R\rho)B},$$
$$G = \frac{\theta B}{R}, \qquad \rho = \frac{\theta(1-\beta)}{R(2-\alpha-\beta)}.$$

For the GBMS with parameters $(R, \theta, \alpha, \beta)$, the PF eigenvalue $\hat{\delta}_{R,\theta,\alpha,\beta}(z)$ is given by

$$\hat{\delta}_{R,\theta,\alpha,\beta}(z) = (\hat{\delta}^*_{\theta,\alpha,\beta}(z))^{1/R}, \quad (4)$$

where $\hat{\delta}^*_{\theta,\alpha,\beta}(z)$ is the PF eigenvalue of $\hat{\boldsymbol{A}}^*_{\theta,\alpha,\beta}(z)$ and is given by (2).

B. Dispersed source model

Now we consider a dispersed source model which is generated from an original GBMS by traffic dispersion. As shown below, the dispersed source model is also described by a GBMS but the parameters of the GBMS is different from those of the original GBMS.

Suppose that traffic generated by a GBMS is spread over d paths, i.e., the dispersion factor is equal to d. From a view of modeling, this situation can be considered as follows: with random order traffic dispersion, a GBMS feeding into a link is virtually decomposed into d identical mini-GBMSs feeding into d paths, and the relation between the parameters (R, θ, α, β) of the original GBMS and the parameters ($R^{(d)}$, $\theta^{(d)}$, $\alpha^{(d)}$, $\beta^{(d)}$) of the mini-GBMS is expressed as

$$R^{(d)} = R, \quad (5)$$

$$\theta^{(d)} = \theta/d, \quad (6)$$

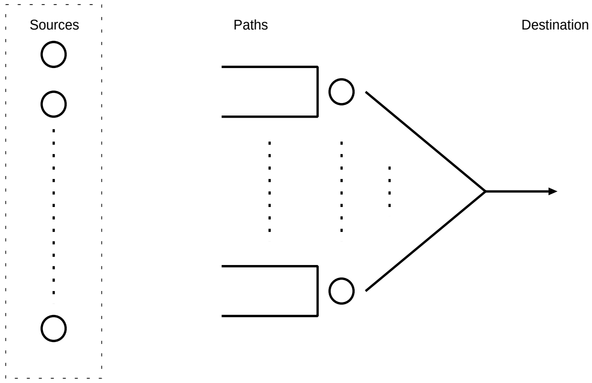

Fig. 2. Network model

$$\alpha^{(d)} = \alpha, \tag{7}$$

$$\beta^{(d)} = \beta. \tag{8}$$

Note that if we set the parameters of the mini-GBMS as (5), (6), (7) and (8), the mean burst length of the mini-GBMS is equal to that of the original GBMS, the sum of the average rates of the d mini-GBMSs is equal to the average rate of the original GBMS, and the sum of the mean burst sizes generated by d mini-GBMSs is equal to the mean burst size generated by the original GBMS. Also note that when $d = 1$, the mini-GBMS is identical to the original GBMS. In other words, the case where traffic dispersion is not used can be considered as the special case where traffic dispersion is used with $d = 1$. Thus, without loss of generality, we hereafter consider only cases where traffic dispersion is employed.

C. Queueing model

In this subsection, we describe a queueing model considered in this paper.

We suppose the following scenario for dispersion.

- As shown in Fig. 2, there are L paths which do not share any physical links, and all paths are homogeneous.
- There are J GBMSs in the network and all the GBMSs are homogeneous in their statistical characteristics.
- Given that $d \le L$, the lth paths ($l = 1, \ldots, Jd - (\lceil Jd/L \rceil - 1)L$) accommodates $\lceil Jd/L \rceil$ mini-GBMSs and the lth paths ($l = Jd - (\lceil Jd/L \rceil - 1)L + 1, \ldots, L$) accommodates ($\lceil Jd/L \rceil - 1$) mini-GBMSs, where $\lceil x \rceil$ denotes the smallest integer that is greater than or equal to x.

For the queueing models corresponding to paths, we assume that packet loss does not occur, and the service time of packet is deterministic and equal to one slot. Let $\{X_n^{(d,l)}\}_{n=0}^{+\infty}$ denote a stochastic sequence representing the queueing process on the lth path when the dispersion factor is equal to d. Its evolution is then described by

$$X_{n+1}^{(d,l)} = (X_n^{(d,l)} - 1)^+ + A_n^{(d,l)}, \tag{9}$$

where $A_n^{(d,l)}$ is a random variable representing the number of total packets arriving from sources to the lth path in the nth slot when the dispersion factor is equal to d. We assume that $\{X_n^{(d,l)}\}_{n=0}^{+\infty}$ is stationary for all d and l.

III. Asymptotic analysis of the queueing model

In this section, we present a formula to evaluate the asymptotic tail distribution of the queue length in the queueing model described in the previous section.

Using the results obtained by the asymptotic analysis [6], [8], [9], [12], we readily obtain the following proposition.

Proposition 1. *Under some conditions (see [6], [8], [9]), for a sufficiently large K and $d \le L$, the tail distribution $\Pr(X_n^{(d,l)} > K)$ of the steady state queue length is asymptotically expressed as*

$$\begin{aligned}\Pr(X_n^{(d,l)} > K) &= c^{(d,l)}(z^{(d,l)})\, e^{-K \log z^{(d,l)}} \\ &\quad + o\left((z^{(d,l)})^{-K}\right),\end{aligned} \tag{10}$$

where $z^{(d,l)}$ is the minimum real solution of $z - \delta^{(d,l)}(z) = 0$ for $z \in (1, \infty)$ and $\delta^{(d,l)}(z)$ is given by

$$\delta^{(d,l)}(z) = \begin{cases} \hat{\delta}_{R^{(d)},\theta^{(d)},\alpha^{(d)},\beta^{(d)}}(z)^{\lceil Jd/L \rceil} \\ \quad \text{for } l = 1, \ldots, Jd - (\lceil Jd/L \rceil - 1)L, \\ \\ \hat{\delta}_{R^{(d)},\theta^{(d)},\alpha^{(d)},\beta^{(d)}}(z)^{\lceil Jd/L \rceil - 1} \\ \quad \text{for } l = Jd - (\lceil Jd/L \rceil - 1)L + 1, \ldots, L, \end{cases}$$

$\hat{\delta}_{R^{(d)},\theta^{(d)},\alpha^{(d)},\beta^{(d)}}(z)$ is obtained by evaluating (4) with parameters $(R^{(d)}, \theta^{(d)}, \alpha^{(d)}, \beta^{(d)})$ and $c^{(d,l)}(z)$ is some function (see [6], [8], [9]).

Remark 1. The constants $c^{(d,l)}(z^{(d,l)})$ and $\log z^{(d,l)}$ appear in (10) are called the asymptotic decay constant and the asymptotic decay rate, respectively. In many studies, $\Pr(X_n^{(d,l)} > K)$ is approximated as

$$\Pr(X_n^{(d,l)} > K) \approx e^{-K \log z^{(d,l)}} = (z^{(d,l)})^{-K}. \tag{11}$$

This approximation thus assumes that $c^{(d,l)}(z^{(d,l)}) = 1$. Further, in many studies, by identifying the loss probability in a finite-buffer queue with the tail distribution of the queue length in the corresponding infinite-buffer queue, the loss probability $P_{\text{loss}}^{(d,l)}$ is approximated as $P_{\text{loss}}^{(d,l)} \approx e^{-K \log z^{(d,l)}} = (z^{(d,l)})^{-K}$, where K denotes the buffer size of the finite-buffer queue.

IV. Numerical results

In this section, we provide numerical results to examine the effect of traffic dispersion on the queueing performance. In the numerical results provided in this section, we assume that the tail distribution $\Pr(X_n^{(d,l)} > K)$ of the queue length on each path is well approximated by (11), the transmission rate on each path is equal to 150Mbps and the size of packet is equal to 512byte.

Fig. 3 shows the tail distribution $\Pr(X_n^{(d,1)} > 40)$ as a function of the dispersion factor d. We set the number of GBMSs and the number of paths as $J = 250$ and $L = 10$, respectively. In Case 1, the parameters of the GBMSs are set as $R = 10$, $\theta = 1.00$, $\alpha = 0.990$ and $\beta = 0.998$. The characteristic of the GBMSs is then as follows: the peak rate

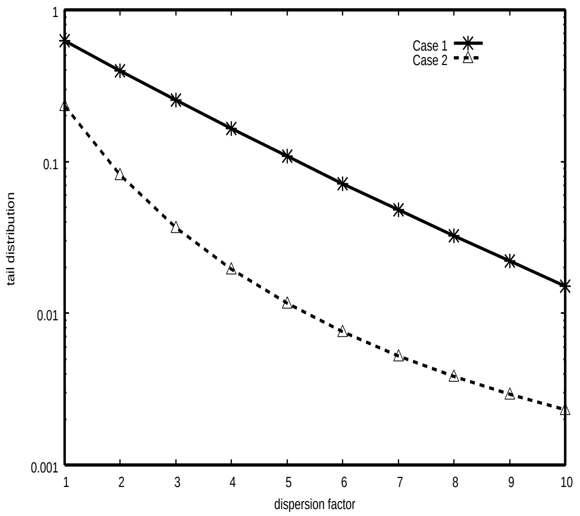

Fig. 3. Effect of dispersion factor on tail distribution

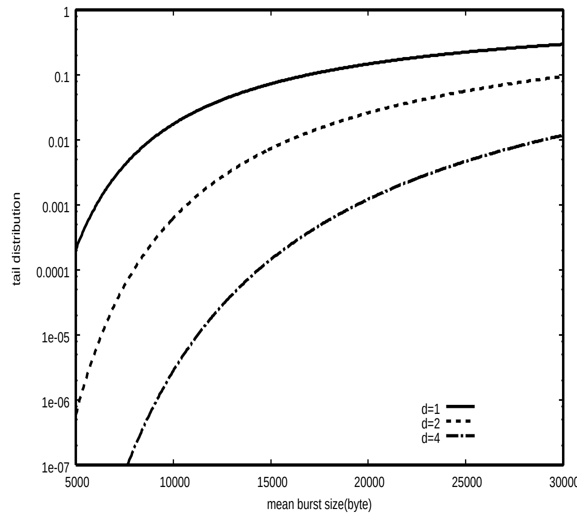

Fig. 4. Effect of mean burst size on tail distribution

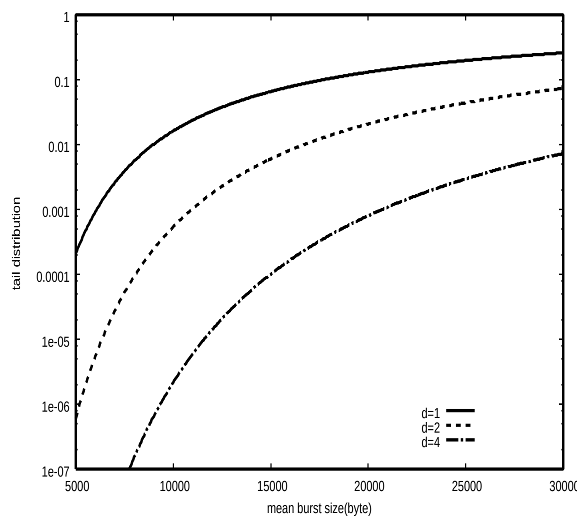

Fig. 5. Effect of mean burst size on tail distribution

is 15.0Mbps, the average rate 2.50Mbps, the mean burst length 2.73×10^{-2}sec, and the mean burst size 5.12×10^{4}byte. In Case 2, the parameters of the GBMSs are set as $R = 10$, $\theta = 1.00$, $\alpha = 0.890$ and $\beta = 0.940$. The characteristic of the GBMSs is then as follows: the peak rate is 15.0Mbps, the average rate 5.29Mbps, the mean burst length 2.48×10^{-3}sec, and the mean burst size 4.65×10^{3}byte. The GBMSs in Case 1 is then more bursty than those in Case 2.

In Fig. 3, we observe the following. Traffic dispersion can reduce the tail distribution of the queue length and improve the queueing performance of packets. The tail distribution of the queue length decreases with the increase in the dispersion factor. The decreasing speed of the tail distribution depends on the statistical characteristics of the sources. For the sources with strong burstiness (i.e., Case 1), the tail distribution decreases almost geometrically within the range of $d = 1, \ldots, 10$. For the sources with weak burstiness (i.e., Case 2), the tail distribution first decreases rapidly, but the decreasing speed of the tail distribution gradually becomes slow with the increase in the dispersion factor. Thus, there is a possibility that a certain “floor” against performance improvement with traffic dispersion exists. When this floor is reached, a further increase in the dispersion factor does not significantly reduce the total effective bandwidth. Simulation results provided in [5] report that for MMPP (Markov Modulated Poisson Process) sources, such a floor is present against cell loss probability.

Next we observe how the mean burst size of the sources affects the performance improvement with traffic dispersion. Figs. 4 and 5 display the tail distribution $\Pr(X_n^{(d,1)} > 40)$ for $d = 1, 2, 4$ as a function of the mean burst size of the original GBMS. We set the number of GBMSs and the number of paths as $J = 80$ and $L = 4$, respectively. In Fig. 4, while fixing $R = 10$, $\theta = 1.00$ and $\beta = 0.998$, we change the mean burst size. The values of α and ρ are then varied with the change in the mean burst size in Fig. 4. On the other hand, in Fig. 5, while fixing $R = 10$, $\theta = 1.00$ and $\rho = 2.00 \times 10^{-2}$, we change the mean burst size. Thus, in Fig. 5, the value of β as well as the value of α is varied with the change in the mean burst size. We observe the following in Figs. 4 and 5. Traffic dispersion can considerably improve the queueing performance, and the improvement effect with dispersion is affected by the mean burst size.

Finally, we consider the asymptotic decay rate ratio $\log z^{(d,l)} / \log z^{(1,l)}$ as the measure of performance improvement in the queueing performance, and we observe how the statistical characteristics of the sources affect the measure of the performance improvement with traffic dispersion. From Remark 1, the asymptotic decay rate ratio $\log z^{(d,l)} / \log z^{(1,l)}$ might be considered as the ratio of the decay speed of the tail distribution of the queue length with the increase in K when the dispersion factor is equal to d to the decay speed when traffic dispersion is not used. If the asymptotic decay rate ratio $\log z^{(d,l)} / \log z^{(1,l)}$ is greater than one, performance

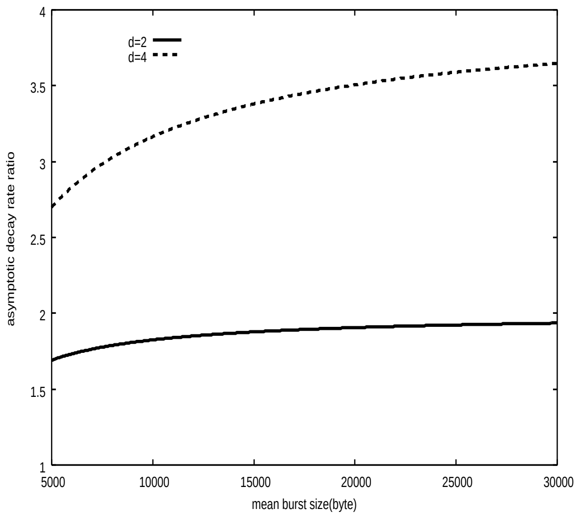

Fig. 6. asymptotic decay rate ratio vs. mean burst size

improvement (i.e., the reduction of the probability that the queue length is greater than some value K) with traffic dispersion will be achieved. Also, if the asymptotic decay rate ratio is large, it is expected that the queueing performance of packets is greatly improved with traffic dispersion.

Fig. 6 shows the asymptotic decay rate ratio $\log z^{(d,1)}/\log z^{(1,1)}$ for $d = 2,4$ as a function of the mean burst size of the original GBMS. The setting in Fig. 6 is the same in Fig. 5. In Fig. 6, we observe that the effect of the performance improvement with traffic dispersion becomes greater with the increase in the mean burst size of the sources.

V. Conclusion

In this paper, we numerically study the queueing performance improvement with traffic dispersion. In particular, we investigate the effect of random order dispersion on the tail distribution of the queue length on a path. In the numerical results, we observe that traffic dispersion makes it possible to improve the queueing performance of packets. The effect of the performance improvement becomes greater with the increase in the dispersion factor and the increase in the mean burst size of the sources.

References

[1] H. Adiseshu, G. Parulkar and G. Varghese, “A reliable and scalable striping protocol,” *ACM Comp. Commun. Rev.*, vol.26, pp.131–141, 1996.

[2] A. Banerjea, “Simulation study of the capacity effects of dispersity routing for fault tolerant realtime channels,” *ACM Comp. Commun. Rev.*, vol.26, pp.194-205, 1996.

[3] S. K. Biswas, R. Izmailov and B. Sengupta, “Connection splitting: an efficient way of reducing call blocking in ATM,” *IEEE/ACM Trans. Networking*, vol.8, pp.655–666, 2000.

[4] Q.-L. Ding and S. C. Liew, “A performance analysis of a parallel communications scheme for ATM networks,” *Proc. of GLOBECOM ’95*, pp.898–902, 1995.

[5] J. H. Déjean, L. Dittmann and C. N. Lorenzen, “String mode - a new concept for performance improvement of ATM networks,” *IEEE J. Sel. Areas Commun.*, vol.9, pp.1452–1460, 1991.

[6] E. Falkenberg, “On the asymptotic behavior of the stationary distribution of Markov chains of M/G/1-type,” *Stoch. Mod.*, vol.10, pp.75–98, 1994.

[7] E. Gustafsson and G. Karlsson, “A literature survey on traffic dispersion,” *IEEE Network*, vol.11, no.2, p.28–36, 1997.

[8] F. Ishizaki and T. Takine, “Bounds for the tail distribution in a queue with the superposition of general periodic Markov sources,” *Proc. of INFOCOM ’97*, pp.1088–1095, 1997.

[9] F. Ishizaki, T. Takine, H. Terada and T. Hasegawa, “Loss probability approximation of a statistical multiplexer and its application to call admission control in high-speed networks,” *Proc. of GLOBECOM ’95*, pp.417–421, 1995.

[10] R. Krishnan and J. A. Silvester, “The effect of multipath routing on the loss performance of multiplexed on-off sources,” *Proc. of International Teletraffic Congress (ITC) 14*, pp.941–950, 1994.

[11] N. F. Maxemchuck, “Dispersity routing in high-speed networks,” *Comp. Networks and ISDN Syst.*, vol.25, pp.645–661, 1993.

[12] K. Sohraby, “On the asymptotic behavior of heterogeneous statistical multiplexer with applications,” *Proc. of INFOCOM ’92*, pp. 839–847, 1992.

[13] C. B. S. Traw and J. M. Smith, “Striping within the network subsystem,” *IEEE Network*, vol.9, no.4, pp.22–29, 1995.